Software to Optimise Hydropower Plant Design

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Abstract

A computer program called HYDRA, that calculates the optimal design parameters of hydropower plants, is described here. This is done by utilizing the well known principle of optimising power losses and construction costs. Up to now it has only been possible to obtain local optima, due to the great complexity of the objective function. The best known example is optimisation of tunnel diameters. The weakness of local optimisation is the substantial risk of missing the true optimum. A new computer model is presented that uses an evolutionary method called Genetic Algorithm to find the global optimal project design by maximising the net profit of power sales.

The model is calibrated against a theoretical example with an optimum that can be obtained by conventional mathematical methods. A case study is performed by comparing the design of an actual project to the optimal results of the model. The most interesting result is that while the model solution produces the same size of power plant, the waterways are different with an overall improvement in economy, clearly showing the importance of global optimisation for the economy of hydropower plants.

Computational System Definition

The basis of the computational system is the Power Planning Model of the Icelandic Energy Authority; called VOS (Þorsteinsson 1985). This is a set of formulas for calculating the investment costs of the various items in hydropower plants, dams, powerhouses, tunnels, and so on.

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Although the system is planned to work on three different planning levels or stages, in this paper we are going to concentrate on the plant stage, comparable to the conventional feasibility level. The other stages, such as the project planning stage, are subjects for further improvements and therefore only briefly introduced here. For complete description of these different levels see Eliasson & Ludvigsson (1996).

The factor controlling the relation between energy production $E$, and necessary installed capacity $N$, is $T_k$, the annual load duration time, and the plant’s load factor, $\lambda_p$, is given as:

$$T_k = \frac{E}{N} \text{[hours]} \quad ; \quad \lambda_p = \frac{T_k}{8760} \quad (1)$$

Mathematical maximisation of an objective function is:

$$\max f(x_1, x_2, \ldots, x_n) \quad \text{for } a_i \leq x_i \leq b_i \text{ for } i = 1 \text{ to } n$$
$$g_j(x_1,x_2,\ldots,x_n) \leq c_j \quad \text{for } \forall j \quad (2)$$

In this chapter the principle of optimal profit is introduced as our objective, so $f(x_1,x_2,\ldots,x_n)$ is the profit, depending on the vector $(x_1,x_2,\ldots,x_n)$, that stores all the necessary variables needed to compute the power production and investment costs. This leads to a method that in fact includes many of the conventional local optimisation methods used so far, and can yield the same results.

By assuming an infinite energy demand and a fixed energy price, $k_e$, the present value of the revenue of energy sale becomes (Eliasson & Ludvigsson 1996):

$$NPV = k_e \cdot E \cdot \left(\frac{1-(1+r)^{-N}}{r}\right) - C \cdot \nu \cdot \left(\frac{1-(1+r)^{-N}}{r}\right) - C \quad (3)$$

where $r$ is the interest rate, $N$ is the lifetime of the investment, $C$ is the project investment, $\nu$ is the operation and maintenance cost, $k_e$ is the unit price of energy, and $E$ is the annual energy capacity of the scheme.

As all costs and revenues are included in the objective function, the optimisation can be considered global. Inserting $NPV$ for $f(x_1, x_2, \ldots, x_n)$ in (2) gives:

$$d \ \NPV = \sum_{i=1}^{n} \frac{\partial \NPV}{\partial x_i} \ dx_i = 0 \ \Rightarrow \ \frac{\partial \NPV}{\partial x_i} = 0 \ \text{for all } i = 1 \text{ to } n \quad (4)$$
The optimisation that Mosonyi (1991), and since then other authors, presents for tunnels, may be deduced from (4). This is local optimisation. Often, variable costs of other project items than the conduit itself, are not taken into account, which results in a larger tunnel diameter than the optimal one.

Calculation of the NPV

The parameters are divided into three main groups; main parameters, local parameters, and coordinates. The main parameters are mutual for all types of structures included in the scheme. From now on the vector \( \overline{MP} = (x_1, x_2, \ldots, x_n) \) indicates these parameters.

The user can define local parameters and coordinates of the nodes, \( P_{i-1} \) and \( P_i \), \( (i = 0 \text{ to } N) \), where the connection points of the structures are located. They are used to calculate sizes such as length of conduits. The nodes can be either fixed or free for optimisation.

The scheme is calculated by starting at the highest point upstream \( (i = 0) \) and ‘send’ the main parameters through each structure \( (j = 1 \text{ to } M) \) on the way downstream to the lowest point \( (i = N) \).

For further explanation, the simple example in Figure 1 is introduced. The dam \( (j = 1) \) is furthest upstream, so \( \overline{MP}_0 = \overline{0} \) is the array entering it. Associated with the dam is some inflow data and a reservoir curve that according to a maximum reservoir level, \( H \), gives the discharge, \( Q_1 \). The height of the dam is a function of the reservoir level, \( H \), and the cost of the dam, \( C_1 \), calculated according to it. The coordinate of the outlet, \( P_1 \), is known so the head produced by the dam is \( H_1 = H - Z_1 \). This gives:

\[
\overline{MP}_1 \cdot Q = Q_1 ; \quad \overline{MP}_1 \cdot H = H_1 ; \quad \overline{MP}_1 \cdot C = C_1
\]

The next structure downstream from the dam is the pressure conduit \( (j = 2) \). For the discharge \( \overline{MP}_1 \cdot Q \) and diameter \( d \) there is a headloss, \( h_f \), in the conduit so the total head produced by the shaft is:

\[
H_2 = \overline{MP}_1 \cdot H + Z_2 - Z_1 - h_f
\]

The discharge in and out is naturally unchanged, and the cost \( C_2 \) is a function of the diameter, lining, tunnelling method, and \( H_2 \). Thus:

\[
\overline{MP}_2 \cdot Q = \overline{MP}_1 \cdot Q ; \quad \overline{MP}_2 \cdot H = H_2 ; \quad \overline{MP}_2 \cdot C = \overline{MP}_1 \cdot C + C_2
\]
Next is the powerhouse \((j = 3)\). It’s main properties are the type and power output of the turbines. Based on \(\bar{MP}_2 \cdot Q\) and \(\bar{MP}_2 \cdot H\), the power \(N_3\) and the energy \(E_3\), is calculated. The cost \(C_3\), is a function of the number of turbines, power capacity, and design head. Discharge is unchanged and the pressure head is reduced to zero by the turbines. This gives:

\[
\begin{align*}
\bar{MP}_3 \cdot Q &= \bar{MP}_2 \cdot Q ; \bar{MP}_3 \cdot H \equiv 0 \\
\bar{MP}_3 \cdot C &= \bar{MP}_2 \cdot C + C_3 ; \bar{MP}_3 \cdot N = N_3 ; \bar{MP}_3 \cdot E = E_3
\end{align*}
\]

A similar procedure is applied to the tailrace. As this is the last structure in this scheme, \(\bar{MP}_4 \cdot C\), \(\bar{MP}_4 \cdot N\), and \(\bar{MP}_4 \cdot E\), are used to calculate the net profit of the investment.

The method used to find the global optimum is called Genetic Algorithm. It is not the purpose of this article to describe the procedure in any detail (see, e.g., Goldberg 1989), but the following main parameters control the process: population size, mutation probability and number of generations.

Each variable in the objective function is subject to the following constraints:
- Each variable, \(p\), must have an upper and lower bound, \(p_{\text{min}} \leq p \leq p_{\text{max}}\). If these bounds are not specified by the user, the program must do so.
- Coordinates that are to be optimised are only allowed to move along a line, not in a three dimensional region.
- The resolution, i.e., the smallest allowed increment of each variable must be defined.
For further explanation, the reader is referred to Eliasson & Ludvigsson (1996).

The Program

The program HYDRA is a 32 bit Windows 95 application, written in Visual Basic 4.0. It includes a SQL connection to the database Access 7.0, where all project data, unit prices, and results, are stored. Also included is a OLE connection to other Windows applications, such as Excel and Word, so results and other data can easily be moved between programs.

When using the computational model, the input process consists of the following steps:
- Define Project
- Assign main assumptions
- Define Structures
- Assign Structure Properties
- Define Points
- Connect Structures
- Edit Unit Prices

When the input phase is finished, the model is ready for calculation. The user can get one detailed solution by using the default variables, see how varying one variable affects the result, or perform a global optimisation. Then different queries can be performed on the [size of population] × [number of generations] size solution set, so the application can, for example, be used for sensitivity and what-if analysis.

Calibration

A ‘simple’ example, much like the one shown in Figure 1, is used to test the validity of the model, and calibrate the parameters of the Genetic Algorithm. For this example, mathematical results can be derived. The mathematics are too long to show here, the results are in Table 1.

When the results of the optimisation are compared to the mathematical solution, it is obvious that the runs where the GA parameters are optimally tuned, reach results very close to the true optimum, see Figure 2. Experience shows that running times lie in the vicinity of 2-4 minutes, depending on the size of population and number of generations.
Table 1. Mathematical solution (bold) compared to optimisation results, NPV, for different number of individuals $P$, generations $G$, and mutation probability $\mu$.

<table>
<thead>
<tr>
<th>$P$</th>
<th>50</th>
<th>50</th>
<th>50</th>
<th>50</th>
<th>50</th>
<th>20</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G$</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.001</td>
<td>0.005</td>
<td>0.01</td>
<td>0.025</td>
<td>0.05</td>
<td>0.025</td>
<td>0.05</td>
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<tr>
<td>$D$</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>3.9</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$H_1$</td>
<td>543.0</td>
<td>543</td>
<td>543</td>
<td>543</td>
<td>543</td>
<td>543</td>
<td>543</td>
</tr>
<tr>
<td>$H_2$</td>
<td>48.2</td>
<td>44</td>
<td>49</td>
<td>49</td>
<td>48</td>
<td>42</td>
<td>50</td>
</tr>
<tr>
<td>$H_3$</td>
<td>44.9</td>
<td>39</td>
<td>46</td>
<td>46</td>
<td>45</td>
<td>37</td>
<td>46</td>
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<tr>
<td>NPV</td>
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<td>28580</td>
<td>28594</td>
<td>28594</td>
<td>28590</td>
<td>28569</td>
<td>28593</td>
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<td>0</td>
<td>0</td>
<td>-4</td>
<td>-25</td>
<td>-1</td>
<td>-18</td>
</tr>
</tbody>
</table>

Figure 2. Development of solution for different number of individuals / mutation probability

The result of the conventional local optimisation method was also calculated and it gave an optimum diameter, $D$, of 4.5 m, which is a 0.5 m difference between methods. This shows how inaccurate the conventional method can be, by ignoring the fact that a change in the diameter not only affects the cost of the conduit but also the cost of, for instance, E & M Equipment and powerhouse. A more detailed description of the difference between the global optimisation and the conventional method is given in Eliasson & Ludvigsson (1996).

Case Study: Fljótsdalur Hydroelectric Project

In cooperation with the National Power Company of Iceland, NPCI, and their engineering consultants, the model was used to perform a case study on the Fljótsdalur Hydroelectric Project. The resulting design variables of the GA optimisation are then compared with the project planning report, PPR, from April 1991 (Eliasson & Ludvigsson 1996). Two runs are made, a Plant Stage run with
VOS formulas, and an Allocation Stage run where the VOS construction cost functions are removed and replaced with new cost functions, specially prepared by the engineering consultants (Helgason, pers. comm.). This is done in order to find out, by comparing the results, if the Allocation Stage could be simulated with the model.

The total drainage area is estimated 478 km², including diversions (Fljótsdalur Engineering Joint Venture 1991). A schematic layout of the project is shown in Figure 3.

Four runs are performed on the Fljótsdalur Hydroelectric Project for the Plant Stage. The results are presented in Table 2, where we have:

- **PPR₁**: The model is calculated for all dimensions fixed according to the PPR.
- **PPR₂**: Same as PPR₁, except the maximum reservoir level of the Eyjabakkar reservoir, which is 4 m higher.
- **O₁₁₅₀**: In order to simulate the effect of a global optimisation on the actual 210 MW scheme, a special run O₁₁₅₀ is introduced, where the maximum energy demand is kept fixed at 1150 GWh/a.
- **O∞**: One feasibility level optimisation is performed with infinite energy demand.

The O₁₁₅₀ optimisation leads to a 0.7 m narrower headrace tunnel compared to the PPR. Local optimisation, considering only variable cost of the headrace, leads to the same result as in the PPR (5 m). The power capacity reduction due to
increased headlosses in the narrower conduits, is compensated by a slightly higher dam (increased discharge to the plant).

Table 2. Significant data and net profit of the investment (optimised dimensions bold). 60 BIKR ≅ 1 billion $

<table>
<thead>
<tr>
<th>Description</th>
<th>PPR₁</th>
<th>PPR₂</th>
<th>O₁₁₅₀</th>
<th>O₁₁₅₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir level m.a.s.l.</td>
<td>664.5</td>
<td>668.5</td>
<td>665.1</td>
<td>667.6</td>
</tr>
<tr>
<td>Headrace tunnel (m)</td>
<td>5.0</td>
<td>5.0</td>
<td>4.3</td>
<td>4.8</td>
</tr>
<tr>
<td>Pressure shaft dia. (m)</td>
<td>2.9</td>
<td>2.9</td>
<td>2.6</td>
<td>2.7</td>
</tr>
<tr>
<td>Power (MW)</td>
<td>213</td>
<td>239</td>
<td>211</td>
<td>233</td>
</tr>
<tr>
<td>Energy (GWh/a)</td>
<td>1159</td>
<td>1300</td>
<td>1150</td>
<td>1278</td>
</tr>
<tr>
<td>Investment (BIKR)</td>
<td>21.16</td>
<td>22.91</td>
<td>19.92</td>
<td>21.96</td>
</tr>
<tr>
<td>Profit (BIKR)</td>
<td>10.90</td>
<td>13.44</td>
<td>12.28</td>
<td>13.86</td>
</tr>
<tr>
<td>∆Profit/∆Investment (%)</td>
<td>0/0</td>
<td>+23/+8</td>
<td>+13/-6</td>
<td>+27/+4</td>
</tr>
</tbody>
</table>

The $O_\infty$ optimisation results in a significantly higher dam compared to the PPR₁. The solution is however not far from the PPR₂ arrangement.

The $O_\infty$ optimisation also results in a slightly smaller headrace diameter compared to the PPR. It is, however, larger than in the $O_{1150}$, which is quite natural.

The project investment is 6% lower in optimisation $O_{1150}$ compared to the PPR₁, resulting in a 13% higher profit, which is a significant improvement. The optimisation $O_\infty$ on the other hand, leads to a 4% higher investment and a 27% higher profit. When it is kept in mind that the PPR₁ plans a future raising of the dam to reservoir level 668.5 m.a.s.l (Fljótsdalur Engineering Joint Venture 1991), the result of $O_\infty$ is very close to the PPR₂ version.

Now the runs as for the Plant Stage are performed with the cost of the whole scheme completely revised. The results are presented in Table 3 (Allocation stage).

Table 3. Significant data and net profit of the investment (optimised dimensions bold). 60 BIKR ≅ 1 billion $

<table>
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<td>Energy (GWh/a)</td>
<td>1159</td>
<td>1300</td>
<td>1149</td>
<td>1325</td>
</tr>
<tr>
<td>Investment (BIKR)</td>
<td>22.78</td>
<td>24.40</td>
<td>22.18</td>
<td>24.91</td>
</tr>
<tr>
<td>Profit (BIKR)</td>
<td>9.36</td>
<td>11.78</td>
<td>9.72</td>
<td>11.97</td>
</tr>
<tr>
<td>∆Profit/∆Investment (%)</td>
<td>0/0</td>
<td>+26/+7</td>
<td>+4/-3</td>
<td>+28/+9</td>
</tr>
</tbody>
</table>
The $O_{1150}$ optimisation leads to a similar arrangement as the plant stage optimisation. The $O_{g165}$, however, shows significant changes. This is because the new cost formulas do not represent the true variation of the costs except in a narrow region around the PPR$_1$ values. Therefore the results of the $O_{g}$ optimisation are hardly applicable. However, a comparison of $O_{g}$ in Tables 2 and 3, shows how important it is to have accurate cost formulas in the optimisation. It may therefore be concluded that it is worth the effort for the consultants, to take the time and trouble to have the cost formulas in Hydra improved with formulas specially designed by themselves, in order to improve the accuracy of optimisations performed.

In such an optimisation, care must be taken to make sure that all the constraints are correct. Otherwise the optimisation can find an unpractical solution, typically tunnel diameters below the minimum required by safe operation. The new dimensions in Table 3 are being checked for this by the owner and the consultants.

Conclusion

The main result of this work is the development of the program HYDRA. The algorithm reaches the true global optimum for the calibration example in less than five minutes, and could undoubtedly be used on a much more complex and detailed objective function. In the Fljótsdalur Hydroelectric Project case optimisation leads to an improved design, with approximately 6% better economical result in the plant stage, due to different dimensions of dam and conduits, and 3% in the allocation stage.

It is our belief, that this new approach makes it possible to globally optimise the design of hydropower projects, with maximum profit as objective, giving better results than the conventional local optimisation methods.

It should be noted that it is not possible, in all cases, to prove that the true global optimum has been reached. However, our experience with the Genetic Algorithm, shows that very good solutions are always reached, if not the optimal one.

Another benefit of this method, is the new possibility to optimise the locations of the structures, as well as their design.

On the other hand, it is also clear that much work has to be done to make this a really powerful tool. The skeleton is formed, but the flesh is missing. The most demanding improvements are:

- The cost functions should be improved, quantity based and connected to an up-to-date price database.
• The model should be expanded to the project planning level.
• Connect the application to a GIS database, to be able to calculate reservoir volumes, dam volumes (varying dam sites), overburden for conduits etc.
• Make the project capable of optimising a series of projects in the same river (Master Plan Studies), instead of optimising single projects.

The conclusion: Promising beginning, long way to go…

Acknowledgements

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